Event shapes, the light-ray OPE, and superconvergence

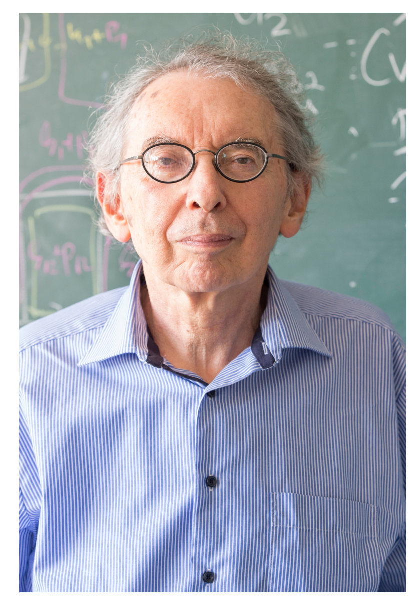
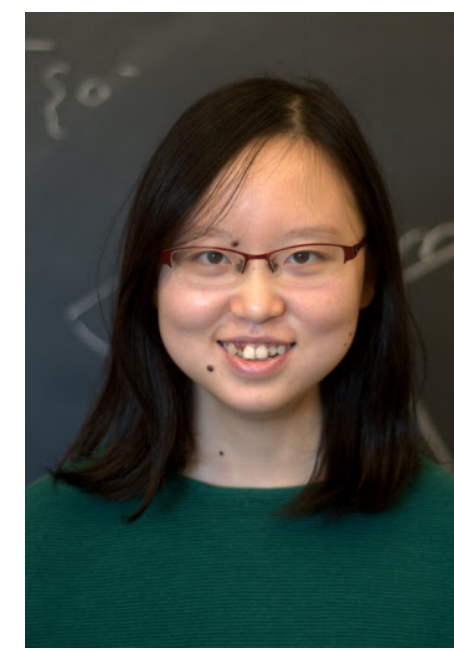
Alexander Zhiboedov, CERN

ITMP, MSU, 2020




based on the recent work with **C-H. Chang, M. Kologlu, P. Kravchuk, D. Simmons-Duffin**



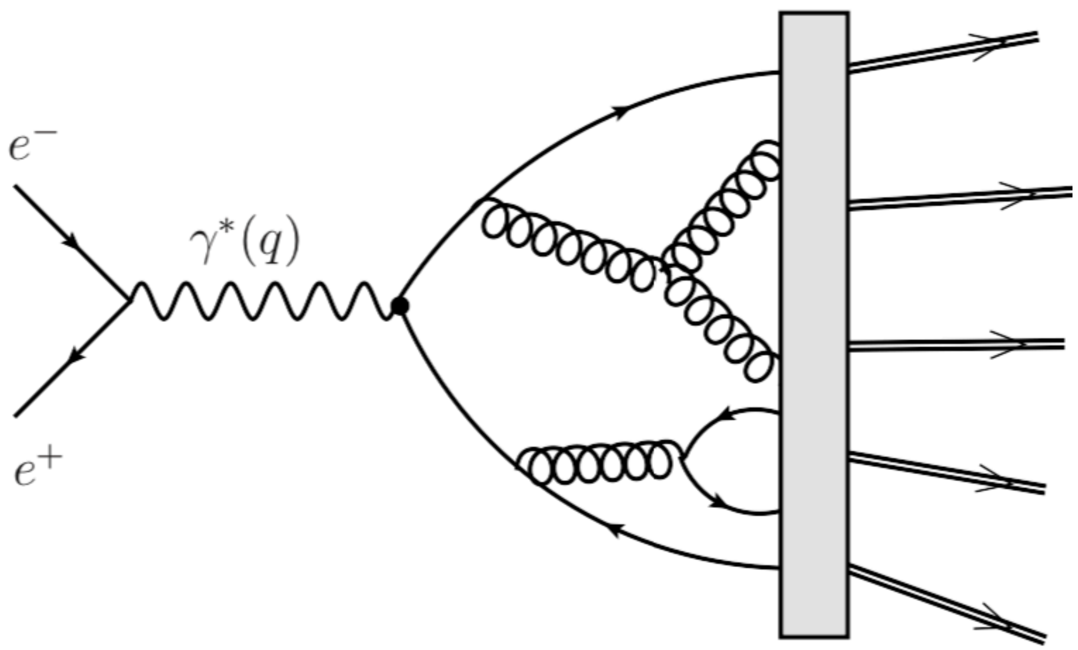




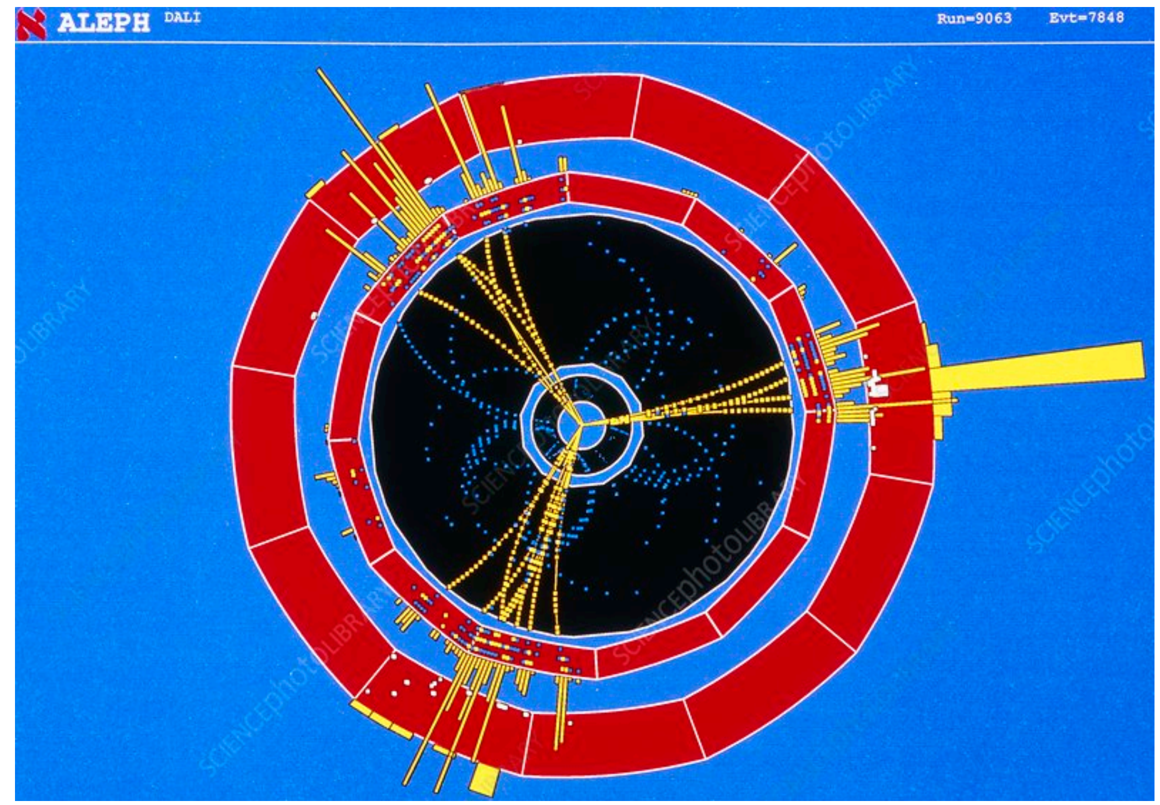
earlier work with: **A. Belitsky, J. Henn, S. Hohenegger, G. Korchemsky, E. Sokatchev, K. Yan**

Introduction

Event shapes

Let us consider annihilation of an electron-positron pair into hadrons





Question: How is the total scattering energy is divided in the final state?

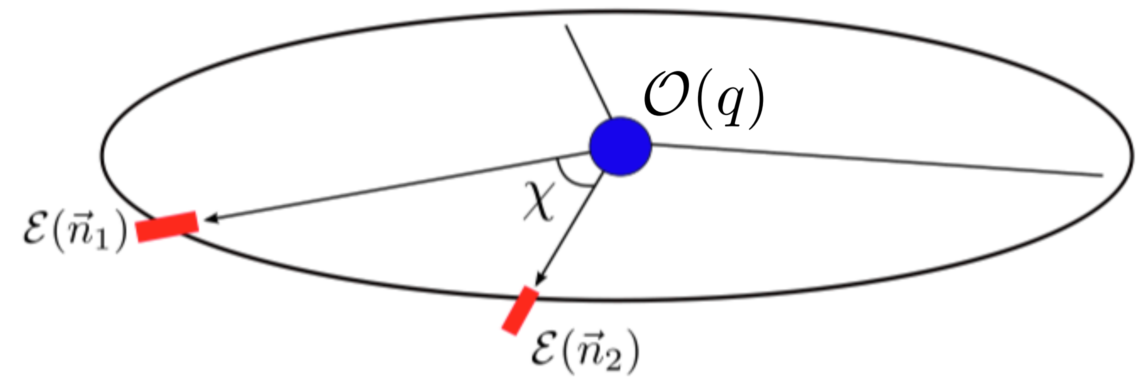
In QCD the final state can be described using a set of infrared&collinear safe observables (**event shapes**): energy-energy correlations, thrust, C-parameter, etc.

Talk: Address this question in the framework of conformal field theories (CFTs).

Energy-energy correlation

$q = (q^0, \vec{0})$

$$\mathrm{EEC}(\chi) = \sum_{a,b,X} \int d\sigma_{a+b+X} \frac{E_a E_b}{(-q^2)} \delta(\cos\theta_{ab} - \cos\chi)$$



[Basham, Brown, Ellis, Love ’78]

- precise experimental data

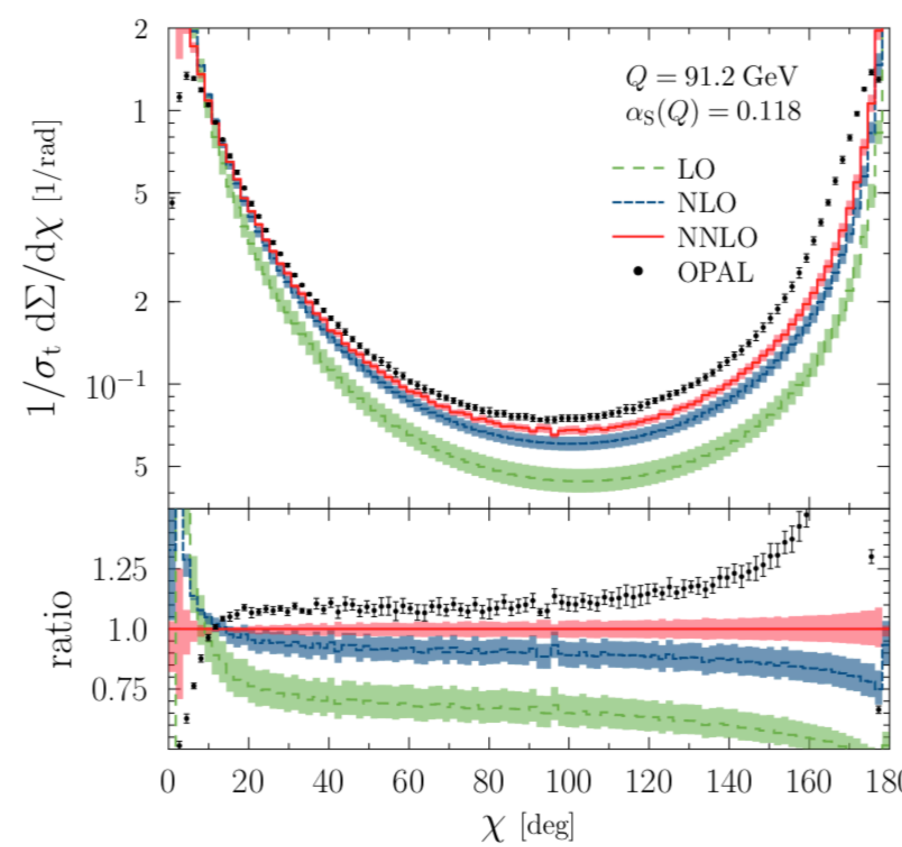
- theory side:

QCD: $\mathrm{EEC}(\chi) = \alpha_s(q) A(\chi) + \alpha_s^2(q) B(\chi) + O(\alpha_s^3)$

40 years!

LO	$\alpha_s(q)$	[Basham, Brown, Ellis, Love ’78]
NLO	$\alpha_s^2(q)$	[Dixon, Luo, Shtabovenko, Yang, Zhu ’18]
NNLO	(numerical)	[Del Duca, Duhr, Kardos, Somogyi, Trocsanyi ’16]

delicate cancellation of the IR divergences



[Tulipánt, Kardos, Somogyi ’17]

What is special about energy correlators?

Operator Language

[Sveshnikov, Tkachov 95′]
[Korchemsky, Sterman 97′]
[Hofman, Maldacena 08′]

The previous definition can be restated in terms of the matrix element of local operators

$$\mathrm{EEC} = \langle \mathcal{O}^\dagger(q)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{O}(q)\rangle$$

where

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{t_0}^{\infty} dt\ n^i T_{0i}(t, r\vec{n})$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\vec{n}^2 = 1$$

acting on the multi-particle state we get

$$\mathcal{E}(\vec{n})|X\rangle = \sum_i k_i^0 \delta^{(2)}(\Omega_{\vec{k}_i} - \Omega_{\vec{n}})|X\rangle$$

- Rethinking jets with energy correlators

[Komiske, Metodiev, Thaler]
[Chen, Moult, Zhang, Zhu '20]

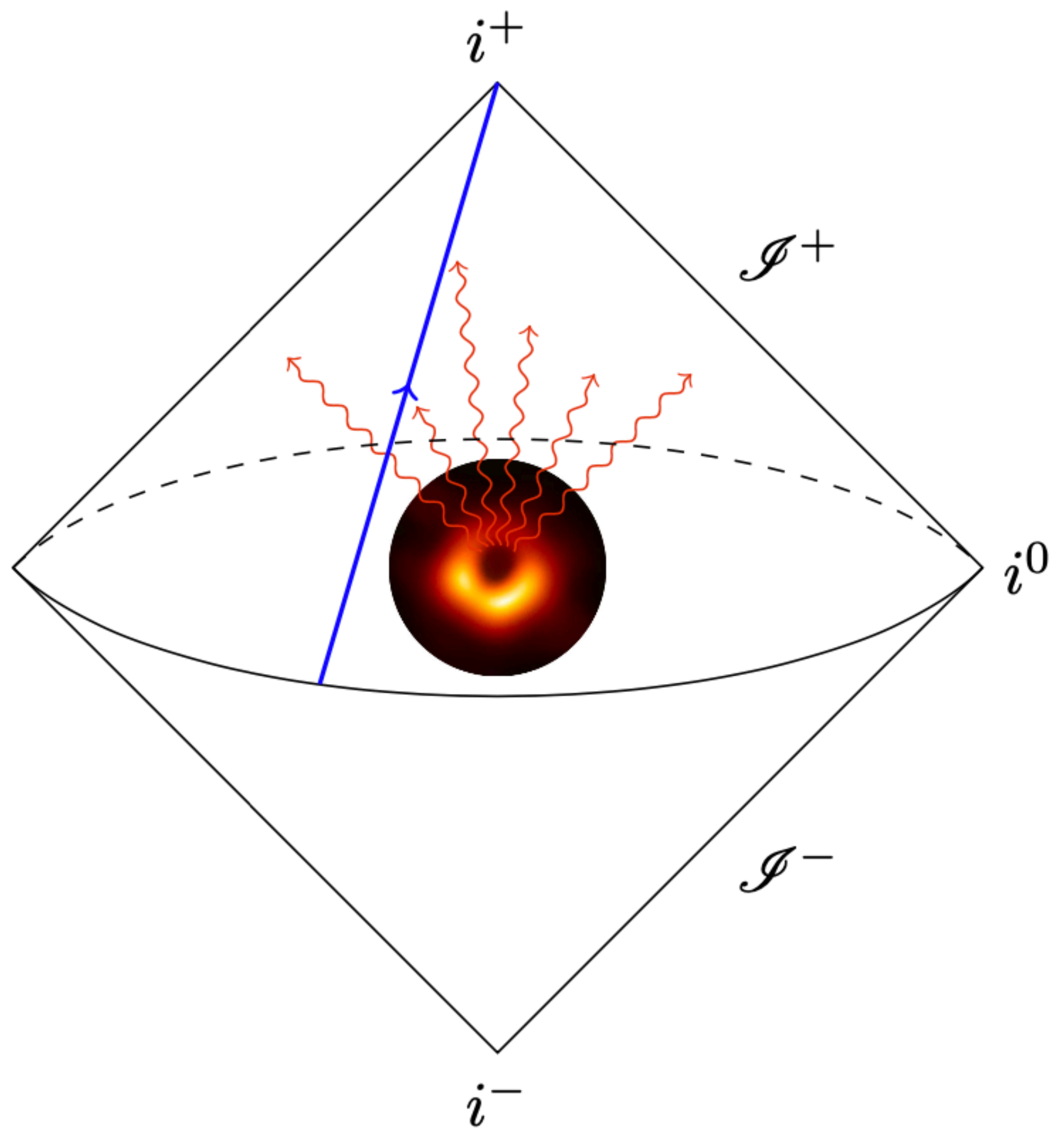
$$\sigma_\omega = \langle \mathcal{O}^\dagger(q)\widehat{\omega}\mathcal{O}(q)\rangle$$

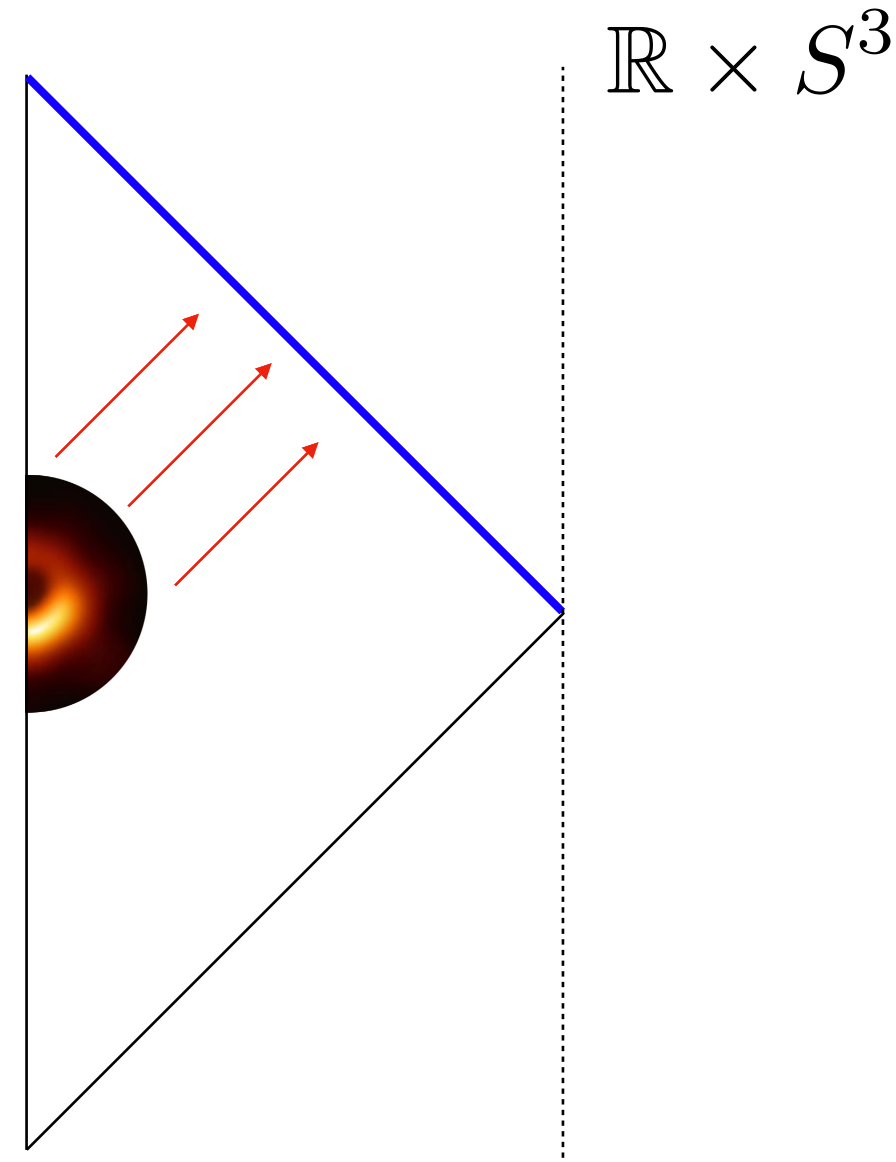
$$\sigma_e = \langle \mathcal{O}^\dagger(q)\delta(e-\widehat{e})\mathcal{O}(q)\rangle$$

$$\int e^n \frac{d\sigma}{de} = \langle \mathcal{O}^\dagger \widehat{e}^n \mathcal{O}\rangle$$

Spacetime picture

Massless (un)particles go to null infinity.





Light-ray operators:
(detectors)

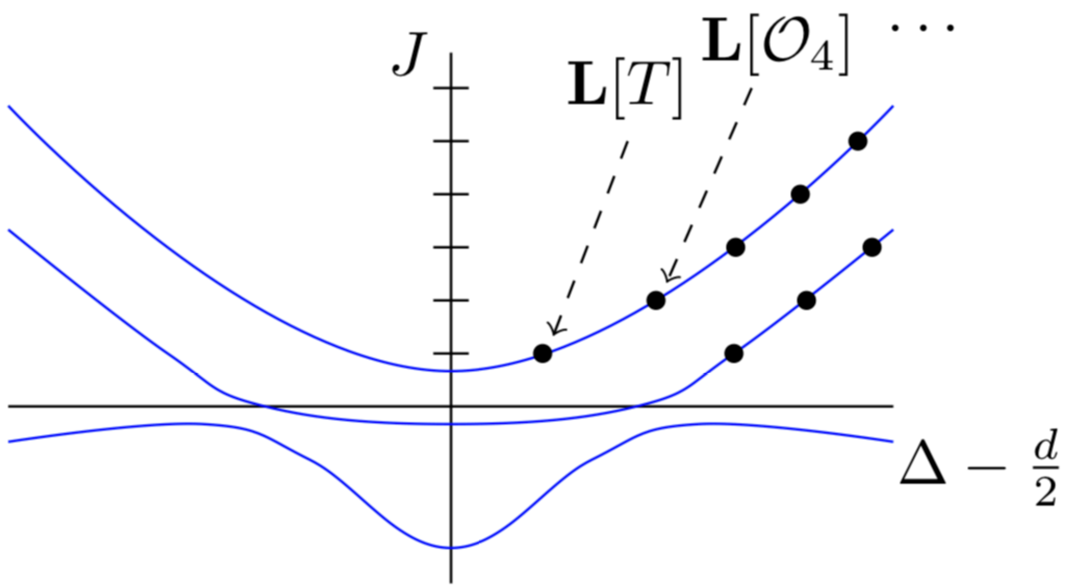
$$\mathcal{E}(\vec{n}) = \mathbf{L}[T] = \int_{-\infty}^{\infty} dv T_{vv}(0, v, \vec{y})$$

Event shapes = matrix elements of light-ray operators

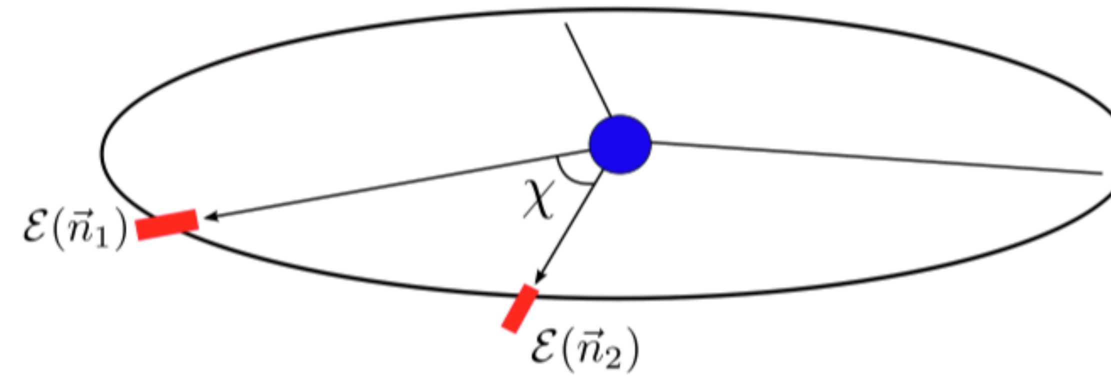
Amplitudes → Event shapes ← **Correlators**

(measurable in collider experiments!)

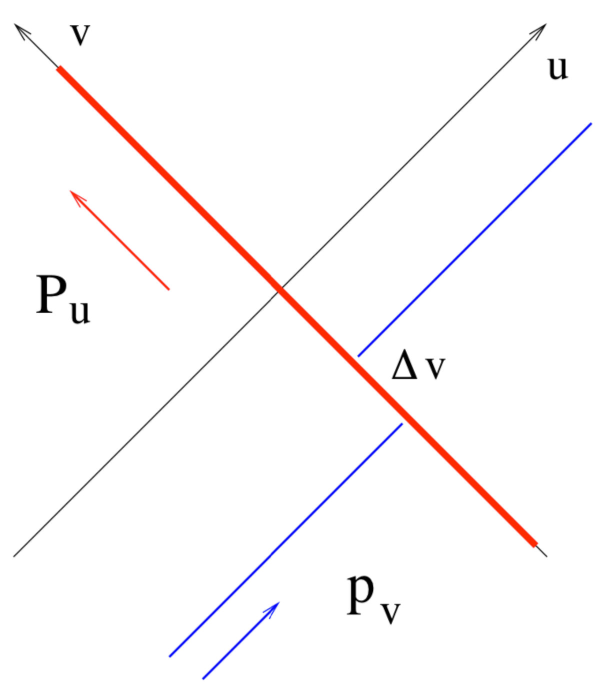
light-ray operators



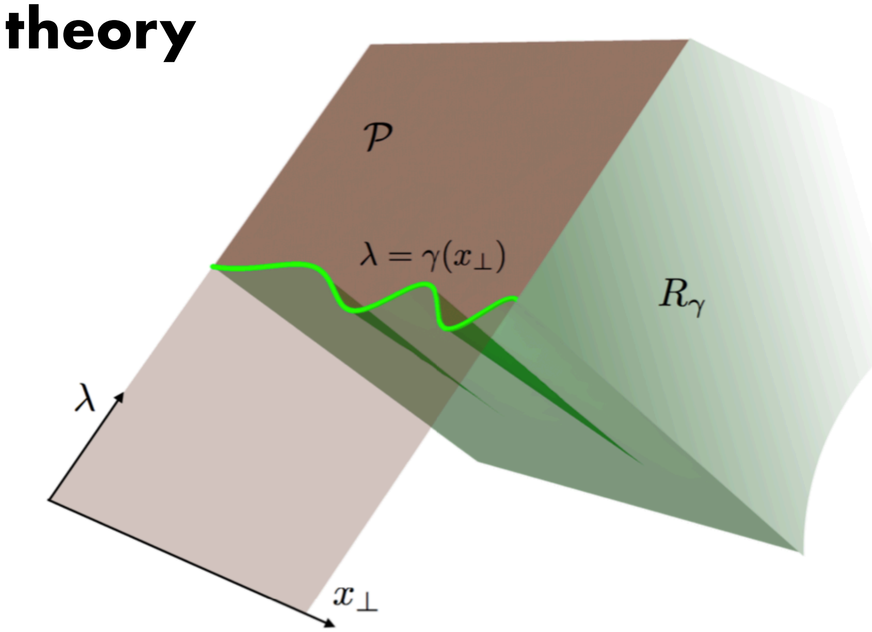
event shapes



gravitational shocks



quantum information theory



Plan of the talk

1. Event shapes in $\mathcal{N} = 4$ SYM

2**. The light-ray OPE**

3**. Superconvergence**

Event shapes in $\mathcal{N} = 4$ SYM

EEC in $\mathcal{N} = 4$ SYM

We consider the stress-energy supermultiplet

$$\{\underbrace{\mathcal{O}_{\mathbf{20'}}}_{\text{scalar}},\ \underbrace{J_{R,\mu}}_{\text{spin one}}\ ,\ \underbrace{T_{\mu\nu}}_{\text{spin two}}\ \}$$

Conformal symmetry and supersymmetry imply

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\Phi(u,v)}{x_{12}^4 x_{34}^4}$$

$$\langle J(x_1)T(x_2)T(x_3)J(x_4)\rangle \sim \underbrace{P(\partial_u,\partial_v)}_{\text{Differential operator}}\ \Phi(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

- $\Phi(u, v)$ is known up to three loops at weak coupling (integrand up to seven loops)
- At strong coupling via the AdS/CFT correspondence

EEC from correlation functions

In practice, to compute the energy-energy correlations

$$\text{EEC} = \underbrace{\int d^4x e^{-iqx}}_{\text{Fourier transform}} \underbrace{\lim_{r_i\to\infty} r_1^2 r_2^2 \int_{t_0}^{\infty} dt_1 dt_2}_{\text{Detector integral}} \underbrace{\langle \mathcal{O}(x) n_1^i T_{0i}(t_1, r_1\vec{n}_1) n_2^j T_{0j}(t_2, r_2\vec{n}_2) \mathcal{O}(0)\rangle}_{\text{Wightman function}}$$

- Compute the Euclidean correlator
- Continue to Minkowski with Wightman prescription
- Perform the detector limit and integral
- Do the Fourier integral

Easy in Mellin space

$$\Phi(u,v) = \int_{-i\infty}^{i\infty} ds dt M(s,t) u^s v^t$$

No IR divergences*, no summing over states, no phase space integrals.

Computationally, in QCD it seems to be much harder than the standard prescription. [Chicherin, Henn, Sokatchev, Yan '20]

However, in CFTs this point of view is quite powerful (and the only possible nonperturbatively)!

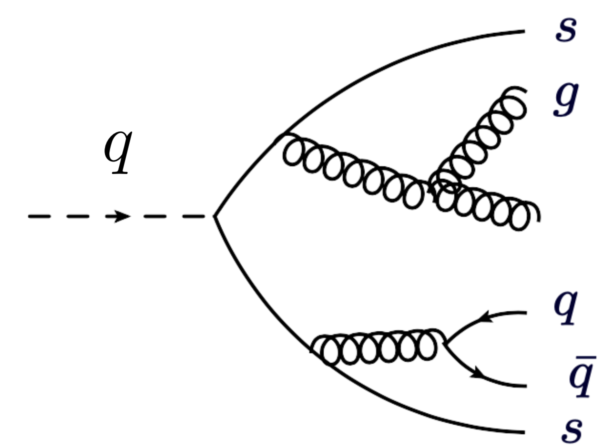
EEC from amplitudes (perturbative)

In a conventional approach we can write

$$\mathrm{EEC} = \sum_{a,b,X} \int d\mathrm{LIPS} |\mathcal{F}_{a+b+X}|^2 E_a E_b \delta(\cos\chi - \cos\theta_{ab})$$

- The relevant building block is a form factor

$$\mathcal{F}_{a+b+X} \equiv \int d^4x e^{iq\cdot x} \langle a, b, X | \mathcal{O}(x) | \Omega \rangle$$



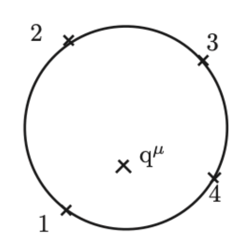
(c) Korchemsky, Talk at ITMP, 2019

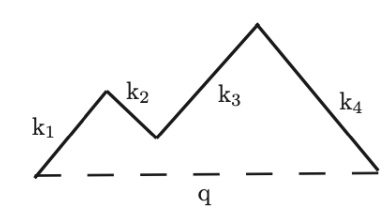
[Bork, Kazakov, Onischenko, Vartanov]
[Brandhuber, Gurdogan, Mooney Spence, Travaglini, Yang]

- Sum over final states

- Integrate over the phase space

- Delicate cancelation of the IR divergences





$$\mathcal{F} = e^{-\mathrm{Area}}$$

impossible at strong coupling!

[Maldacena, AZ]

Data (perturbative)

$$a = \frac{g_{YM}^2 N_c}{4\pi^2}$$

$$\text{EEC}_{\mathcal{N}=4} = \frac{1}{4\zeta^2(1-\zeta)}(aF_1(\zeta) + a^2 F_2(\zeta) + a^3 F_3(\zeta) + \ldots), \quad \zeta = \frac{1-\cos\chi}{2}$$

- **LO** $$F_1(\zeta) = -\log(1-\zeta)$$

$$\omega(\zeta(n)) = n$$
$$\omega(\pi) = \omega(\log) = 1$$

- **NLO**

$$\begin{aligned}
F_2(\zeta) &= 4(1-\zeta)\sqrt{\zeta}\left(\text{Li}_2\left(-\sqrt{\zeta}\right) - \text{Li}_2\left(\sqrt{\zeta}\right) + \frac{1}{2}\log\left(\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\right)\log(\zeta)\right) + \frac{1}{3}\pi^2(1-\zeta)\zeta \\
&+ 2(1-\zeta)\log(1-\zeta)\log\left(\frac{\zeta}{1-\zeta}\right) + \left(1-\zeta^2\right)\left(2\text{Li}_2(\zeta) + \log^2(1-\zeta)\right) \\
&+ \frac{1}{4}\left(6\left(-4\zeta^2+3\zeta+3\right)\text{Li}_3\left(\frac{\zeta}{\zeta-1}\right) + 2\text{Li}_2(\zeta)\left(2\left(2\zeta^2-\zeta-2\right)\log(1-\zeta) + (3-4\zeta)\zeta\log(\zeta)\right)\right. \\
&+ 2\text{Li}_2(\zeta)\left(2\left(2\zeta^2-\zeta-2\right)\log(1-\zeta) + (3-4\zeta)\zeta\log(\zeta)\right) + \frac{1}{3}\pi^2\left(2\zeta^2\log(\zeta) + \left(-2\zeta^2-\zeta+2\right)\log(1-\zeta)\right) \\
&+ (1-\zeta)(2\zeta+1)\left(-8\text{Li}_3\left(\frac{\sqrt{\zeta}}{\sqrt{\zeta}-1}\right) - 8\text{Li}_3\left(\frac{\sqrt{\zeta}}{\sqrt{\zeta}+1}\right) + \log\left(\frac{1-\zeta}{\zeta}\right)\log^2\left(\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}}\right)\right) \\
&\left.+ 6\left(-4\zeta^2+3\zeta+3\right)\text{Li}_3\left(\frac{\zeta}{\zeta-1}\right) - 4(\zeta-4)\text{Li}_3(\zeta) - 2\zeta(4\zeta+1)\zeta(3)\right)
\end{aligned}$$

[Belitsky, Hohenegger, Korchemsky, Sokatchev, AZ '13]

- **NNLO** $$F_3(\zeta) = \text{Harmonic polylogarithms} + \text{Elliptic integral}$$

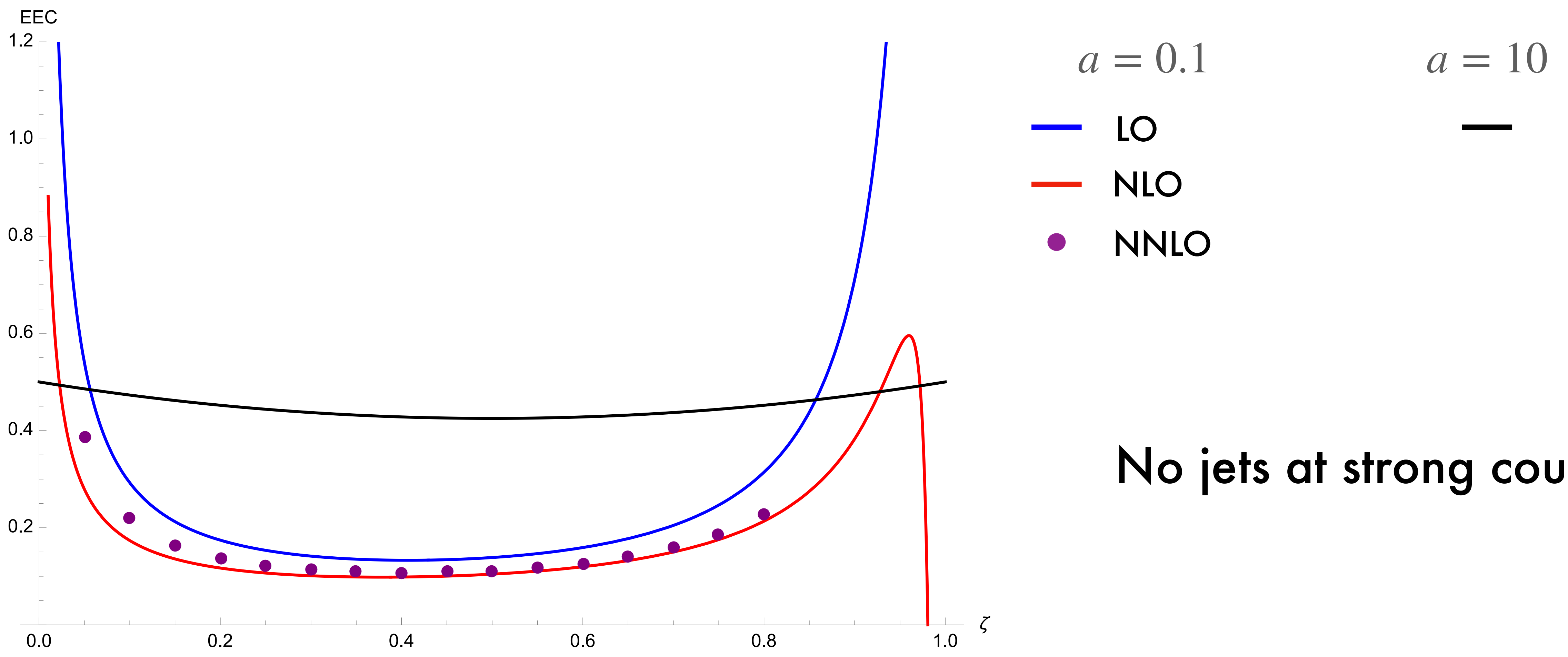
[Henn, Sokatchev, Yan, AZ '19]

- Strong coupling $a \to \infty$

$$\text{EEC}_\infty = \frac{1}{2}\left(1 + \frac{6\zeta^2 - 6\zeta + 1}{a} + \ldots\right)$$

[Hofman, Maldacena '08]

Data



No jets at strong coupling!

- Small angle $\zeta \to 0$

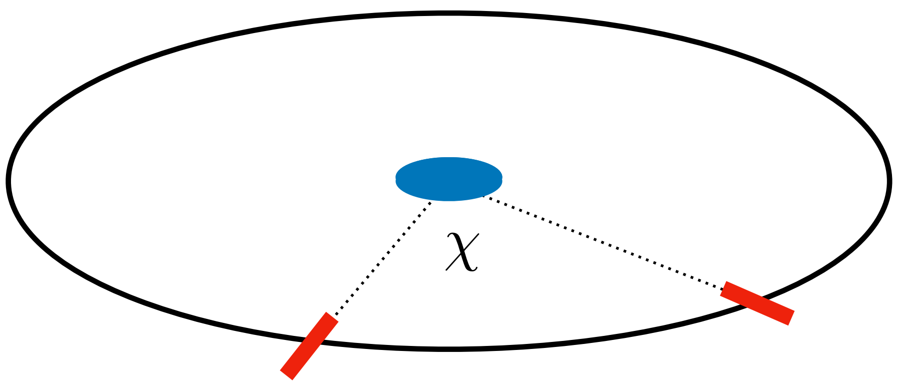
[Dixon, Moult, Zhu '19]
[Korchemsky '19]

$$\mathrm{EEC}_{\mathrm{QCD}}(\chi \to 0) = \mathrm{EEC}_{\mathcal{N}=4}(\chi \to 0) + \text{lower weights}$$

- Back-to-back $\zeta \to 1$

$$\mathrm{EEC}_{\mathrm{QCD}}(\chi \to \pi) = \mathrm{EEC}_{\mathcal{N}=4}(\chi \to \pi) + \text{lower weights}$$

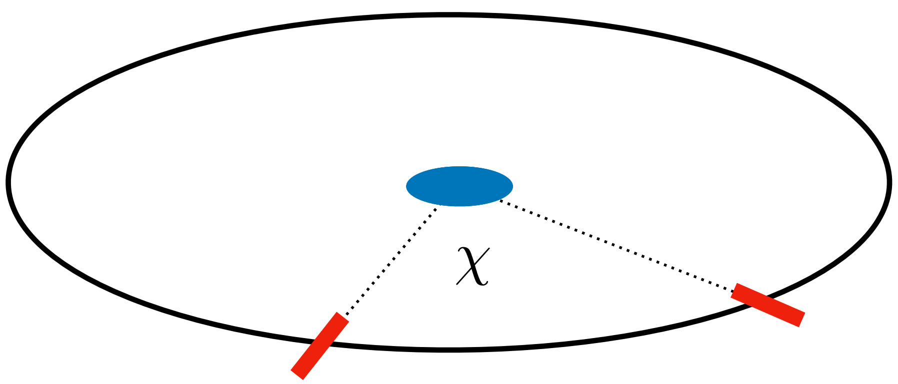
Event picture



Weak coupling (jets):

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{q^0}{\mathrm{vol} S^{d-2}}$$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle \sim \delta(\chi) + \delta(\chi - \pi)$$



Strong coupling (no jets):

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{q^0}{\mathrm{vol} S^{d-2}}$$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle = \left(\frac{q^0}{\mathrm{vol} S^{d-2}} \right)^2$$

$$\langle \mathcal{E}(\vec{n}_1) \ldots \mathcal{E}(\vec{n}_k) \rangle = \left(\frac{q^0}{\mathrm{vol} S^{d-2}} \right)^k$$

How do we interpolate between the two pictures?

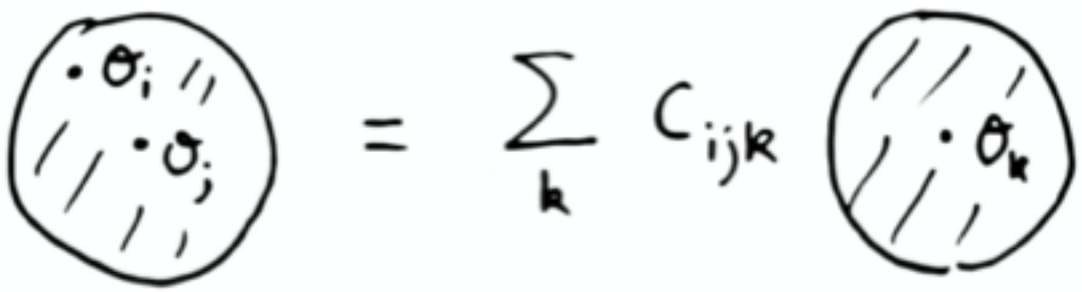
The light-ray OPE

Can we understand these observables nonperturbatively?

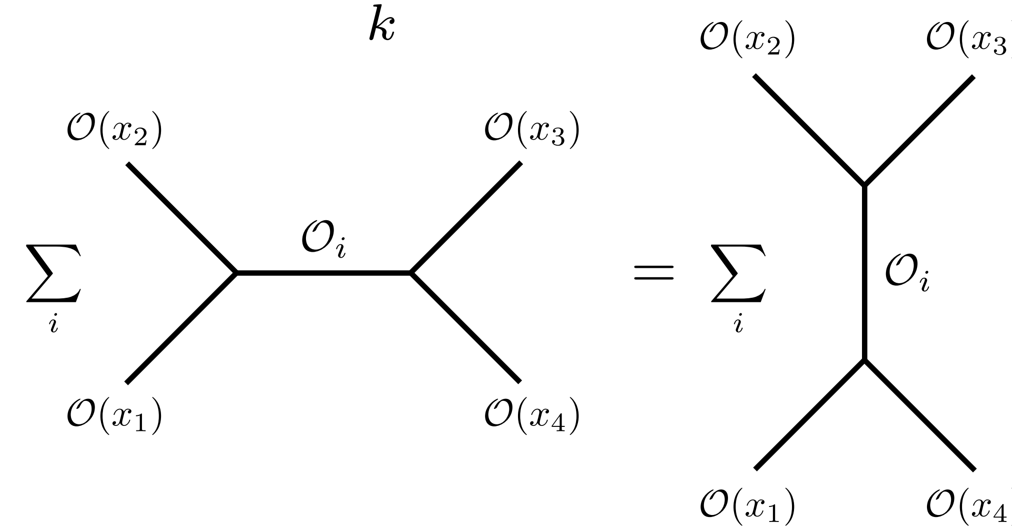
Operator product expansion (OPE)

OPE for local operators is a powerful tool, eg it enables one to formulate the bootstrap equations

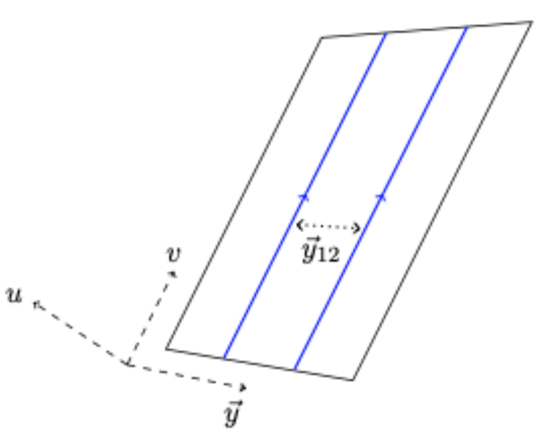
$$\mathcal{O}(x_1)\mathcal{O}(x_2) = \sum_k C_{ijk}(x_{12}, \partial_2)\mathcal{O}_k(x_2)$$

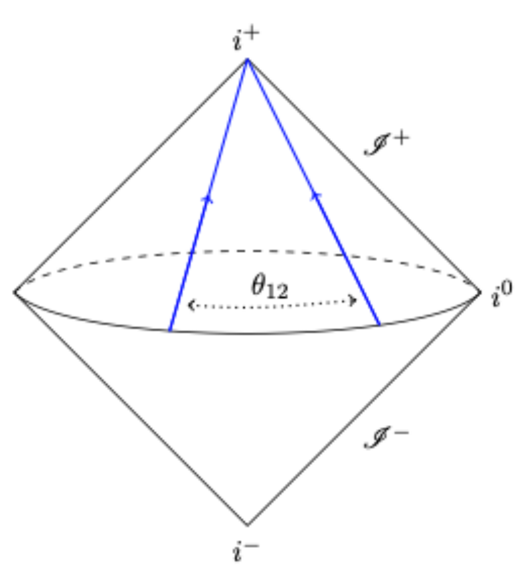


(c) D. Simmons-Duffin



Can we do the same operation for the light-ray operators (celestial energy calorimeters)?





Lightray operator = Local on the celestial sphere

$$\underbrace{\text{Small angles} = \text{Small transverse distance}}_{\text{Stereographic projection}}$$

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} dv T_{vv}(0, v, \vec{y})$$

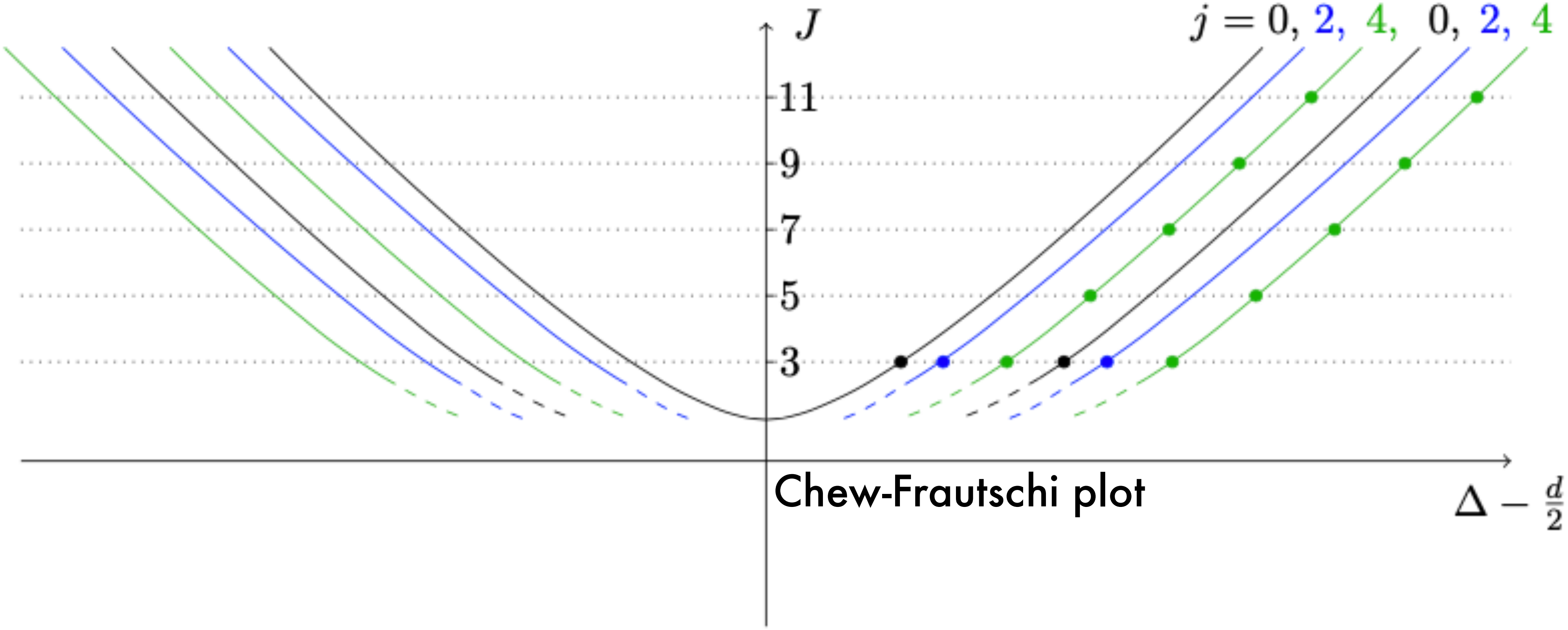
Light-ray OPE

[Hofman, Maldacena '08]
[Kologlu, Kravchuk, Simmons-Duffin, AZ '19]
[Chang, Kologlu, Kravchuk, Simmons-Duffin, AZ '20]

Remarkably, the answer to this question is positive!

$$\mathcal{E} \times \mathcal{E} = \sum_i \left(\mathbb{O}^+_{i,J=3,j=0} + \mathbb{O}^+_{i,J=3,j=2} + \mathbb{O}^+_{i,J=3,j=4} \right) + \sum_{n,i} \mathcal{D}_{2n} \mathbb{O}^+_{i,J=3+2n,j=4}$$

$T_{\mu\nu} \times T_{\rho\sigma}$



$\mathcal{O}_{\Delta,J}$

Δ – scaling dimension

J – spin

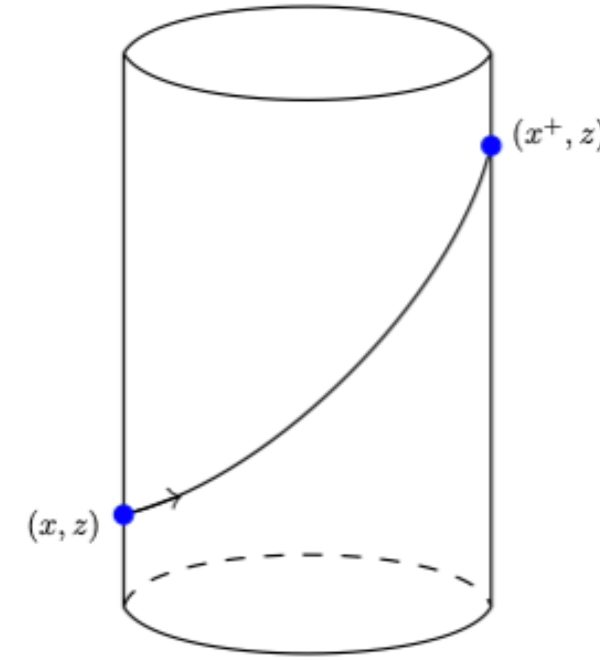
$\Delta^{\pm}(J)$ – Regge trajectory

[Caron-Huot '17]

CFT light-ray operators

[Kravchuk, Simmons-Duffin 18']

Start with a bi-local object



$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) \equiv \pm \int' d^dx_1 d^dx_2 K^t_{\Delta,J}(x_1,x_2;x,z)\phi_1(x_1)\phi_2(x_2)$$
$$+ \int' d^dx_1 d^dx_2 K^u_{\Delta,J}(x_1,x_2;x,z)\phi_2(x_2)\phi_1(x_1)$$

The kernel is obtained by Wick-rotating and light-transforming the conformal partial wave expansion.

The residues are light-ray operators

$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) \sim \frac{1}{\Delta - \Delta^{\pm}_i(J)} \mathbb{O}^{\pm}_{i,J}(x,z)$$

For integer spins

$$\mathbb{O}^{\pm}_{i,J}(x,z) \sim \int_{-\infty}^{\infty} dv \; [\text{local operator}] \begin{matrix} + = \text{even spins} \\ - = \text{odd spins} \end{matrix}$$

Finite Coupling (spinless target)

The lightray-lightray OPE (finite coupling): $\sum_i \mathbb{O}^+_{i,J=3,j=0}$

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle = \frac{(q^0)^2}{8\pi^2}\left[\sum_i p_{\Delta_i}\frac{4\pi^4\Gamma(\Delta_i-2)}{\Gamma(\frac{\Delta_i-1}{2})^3\Gamma(\frac{3-\Delta_i}{2})}f^{4,4}_{\Delta_i}(\zeta)+\frac{1}{4}(2\delta(\zeta)-\delta'(\zeta))\right]$$

spinless target!

$$f^{\Delta_1,\Delta_2}_{\Delta}(\zeta)=\zeta^{\frac{\Delta-\Delta_1-\Delta_2+1}{2}}\,{}_2F_1\left(\frac{\Delta-1+\Delta_1-\Delta_2}{2},\frac{\Delta-1-\Delta_1+\Delta_2}{2},\Delta+1-\frac{d}{2},\zeta\right)$$

$$\zeta=\frac{1-\cos\chi}{2}$$

- Sum is over Regge trajectories
- OPE data is evaluated at **J=3** (p_{Δ_i})
  (three-point functions)
- Agrees with the perturbative results
- Correctly captures contact terms

Ward identities:

$$\int_0^1 d\zeta\ \langle\mathcal{E}\mathcal{E}\rangle=1$$

$$\int_0^1 d\zeta\ (2\zeta-1)\langle\mathcal{E}\mathcal{E}\rangle=0$$

Perturbative Checks

1. Decompose correlator in the OPE
2. Continue each trajectory in spin

One loop, weak coupling

$$\mathrm{EEC}_{\text{1-loop}} = \sum_{n=1}^{\infty} \frac{(n!)^2}{2(2n)!} S_1(n) f^{4,4}_{3+2n}(\zeta) = -\frac{1}{4}\frac{\log(1-\zeta)}{\zeta^2(1-\zeta)}$$

Sum over Regge trajectories

OPE data evaluated at spin $J = 3$

Strong coupling

$$\mathrm{EEC}_{\infty} = \sum_{n=0}^{\infty} \frac{(-1)^n}{8}(n+1)(n+2)(n+3)(n+4)\frac{\Gamma(n+3)^2}{\Gamma(2n+5)} f^{4,4}_{7+2n}(\zeta)$$

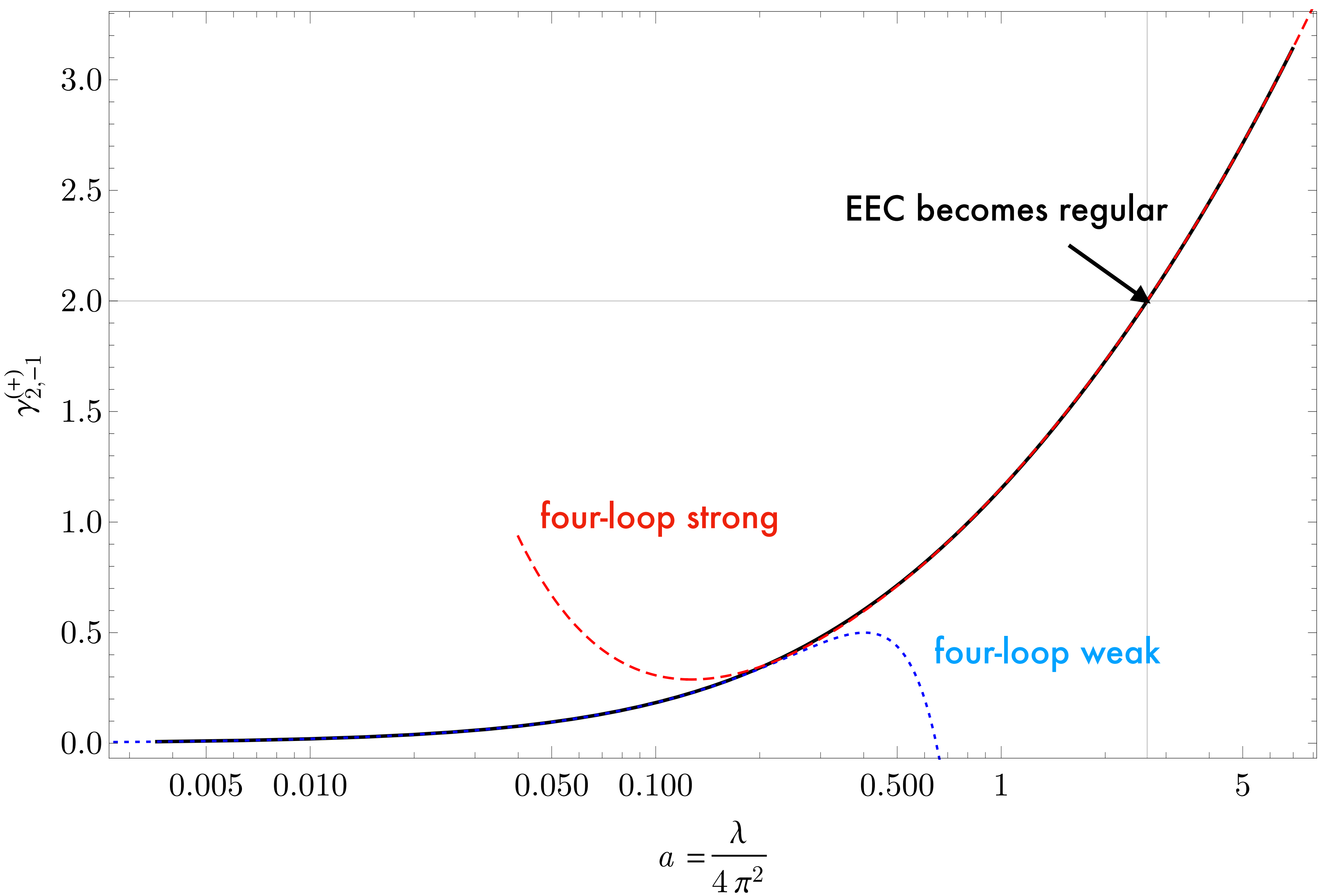
$$= \frac{1}{2}$$

Jets at Finite Coupling in N=4 SYM

[Hofman, Maldacena 08']
[Korchemsky 19']
[Dixon, Moult, Zhu 19']
[Kologlu, Kravchuk, Simmons-Duffin, AZ '19]

$$\langle \mathcal{E}(\theta)\mathcal{E}(0)\rangle \sim \frac{1}{\theta^{2(1-\frac{\gamma^{+}_{2,-1}}{2})}}$$

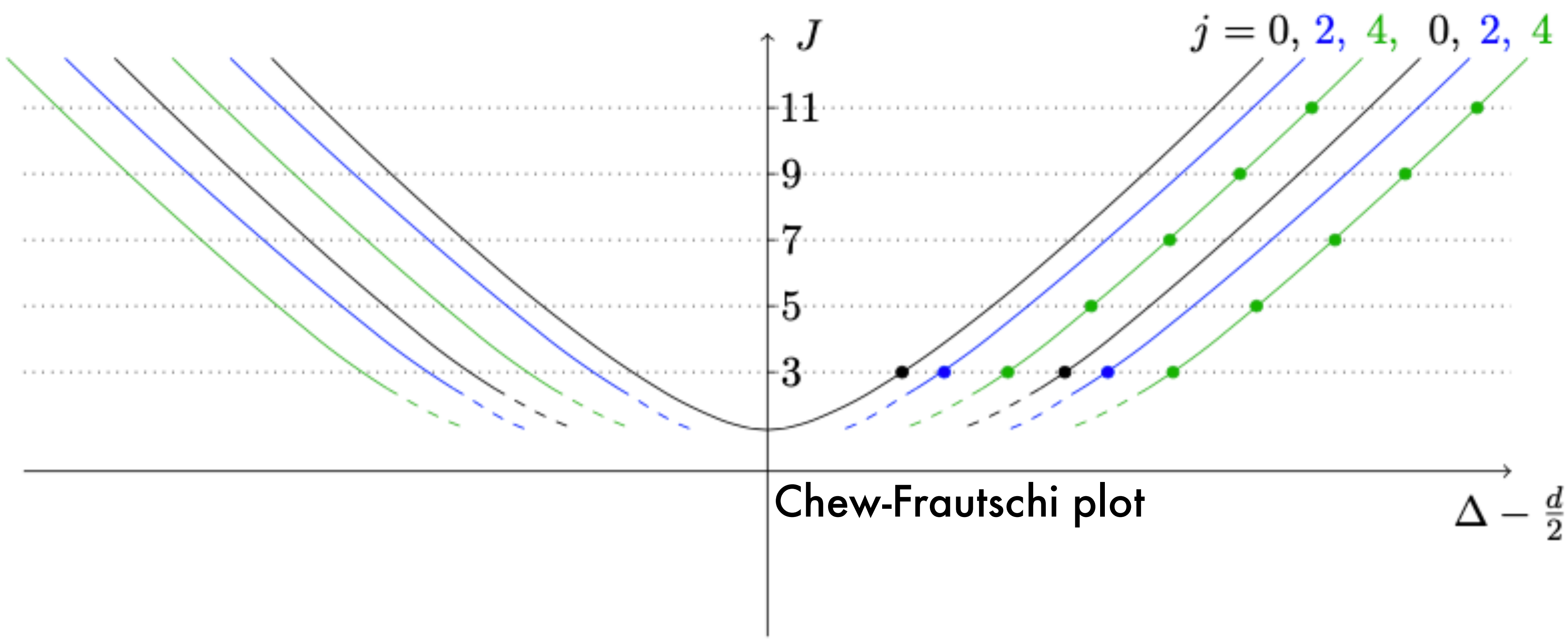
integrability:



[N.Gromov]
[Gromov, Levkovich-Maslyuk, Sizov 15']
[Gromov, Kazakov, Leurent, Volin 13']

What is *j*?

$$\mathcal{E} \times \mathcal{E} = \sum_i \left(\mathbb{O}^+_{i,J=3,j=0} + \mathbb{O}^+_{i,J=3,j=2} + \mathbb{O}^+_{i,J=3,j=4} \right) + \sum_{n,i} \mathcal{D}_{2n} \mathbb{O}^+_{i,J=3+2n,j=4}$$



Chew-Frautschi plot

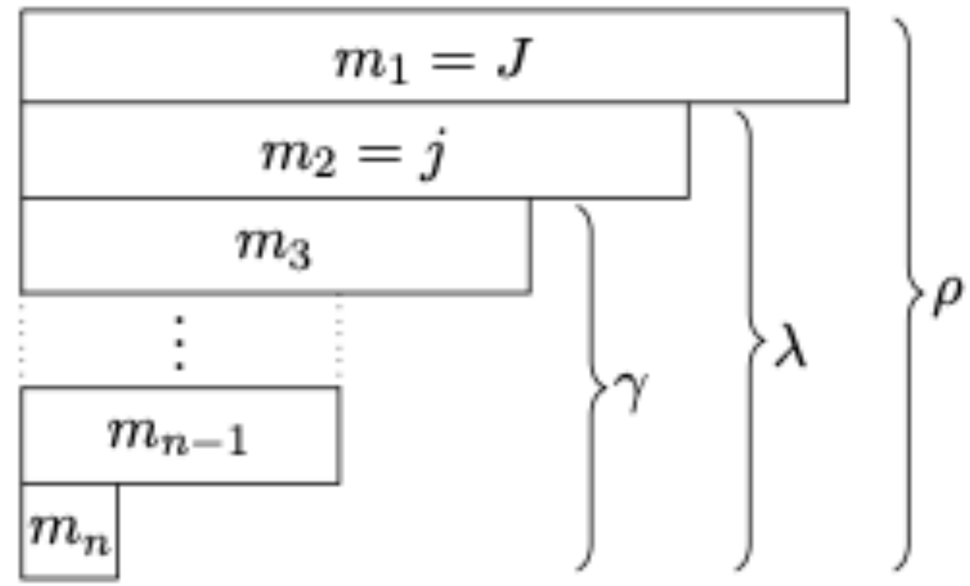
Transverse spin

$$\mathrm{SO}(d-1,1)$$

$$n=\left[\frac{d}{2}\right]$$

SPIN

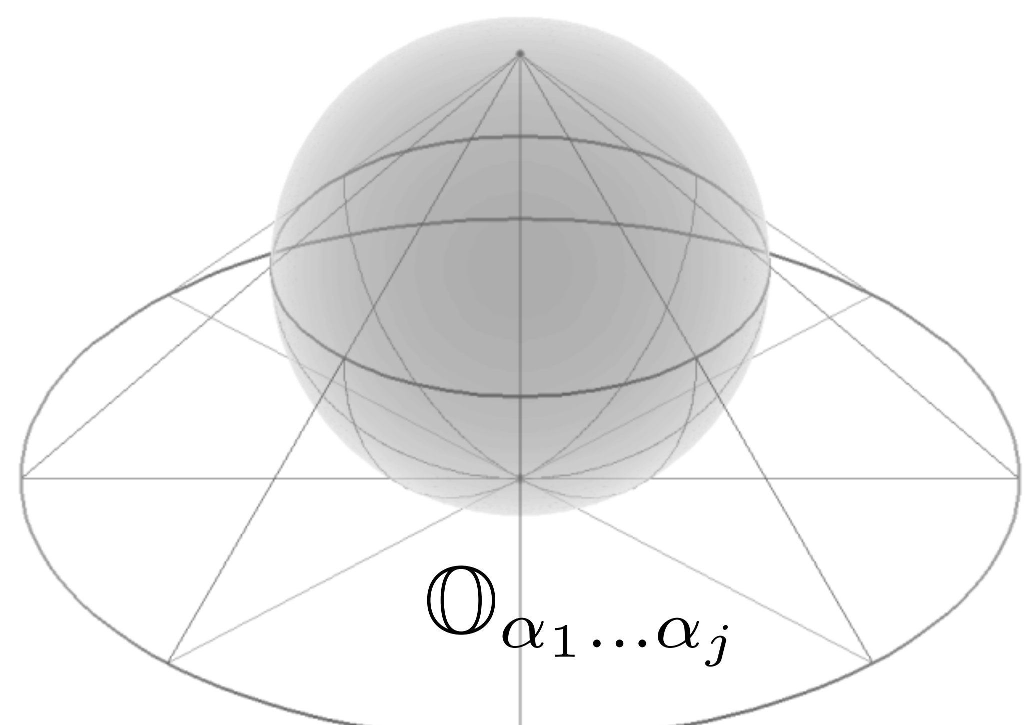
transverse spin



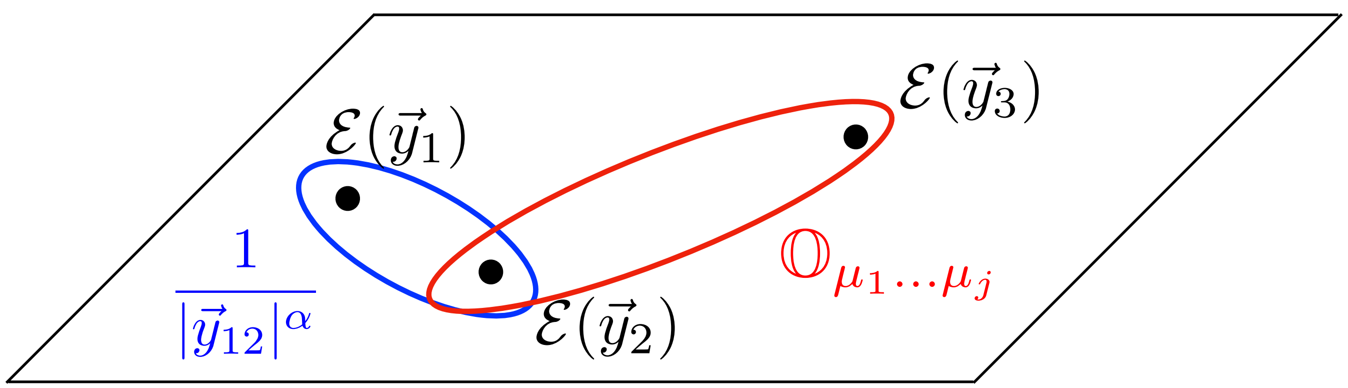
Transverse spin of the light-ray operator:

$$\mathbb{O}_j$$

Celestial sphere



Puzzle and its resolutions



Expectation:

Celestial crossing predicts light-rays of arbitrarily large transverse spin in the OPE!

Puzzle:

$$\mathcal{E} \times \mathcal{E} \stackrel{?}{=} \sum_i \left(\mathbb{O}^+_{i,J=3,j=0} + \mathbb{O}^+_{i,J=3,j=2} + \mathbb{O}^+_{i,J=3,j=4} \right)$$

[Kologlu, Kravchuk, Simmons-Duffin, AZ '19]

Resolution: Primary descendants!

$$+ \sum_{n,i} \mathcal{D}_{2n} \mathbb{O}^+_{i,J=3+2n,j=4}$$

[Chang, Kologlu, Kravchuk, Simmons-Duffin, AZ '20]

$$\mathcal{D}_n : (\underbrace{j+n-1}_{\text{spin}}, \underbrace{j}_{\text{transverse spin}}) \to (j-1, j+n).$$

conformal differential operator

Conformal Differential Operators

$$V_{\Delta,J}: \qquad \text{primary operator } \mathcal{O}, \underbrace{\partial\mathcal{O}, \partial^2\mathcal{O}, \ldots}_{\text{descendants}}$$

This is not always the case! For some very special quantum numbers it can become reducible. For example

$$V_{d-1,1} \supset V_{d,0}$$

Primary descendant: $\partial_\mu J^\mu$

Conserved current (short representation): $\widetilde{V}_{d-1,1} \equiv V_{d-1,1}/V_{d,0}$

$$\partial_\mu J^\mu = 0$$

Conformal differential operators arising in this way are quite special and they has been classified. [Penedones, Trevisani, Yamazaki '15]

Do not play role for local operators due to unitarity bounds:

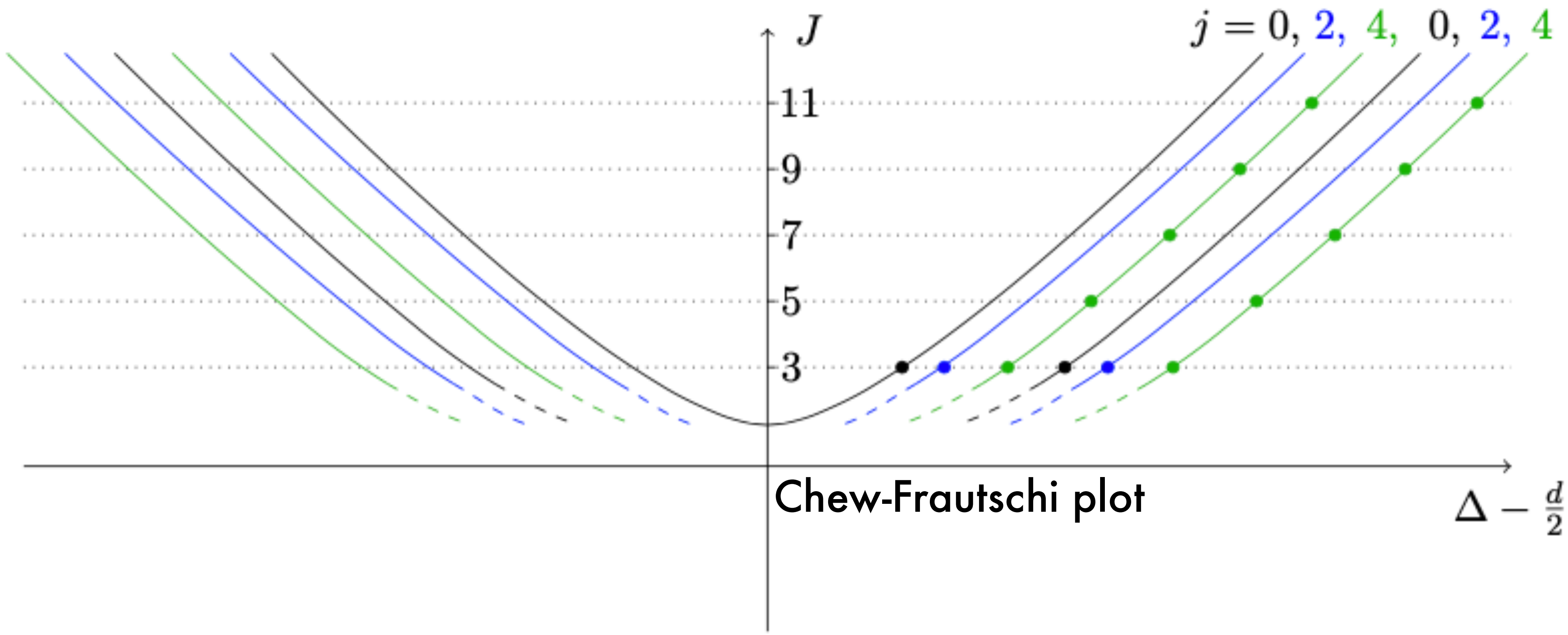
$$\Delta(J) - J \geq d - 2$$

Important for light-ray operators: $\mathbf{L} : (\Delta, J) \to (1-\Delta, 1-J)$

Light-ray OPE

$$\mathcal{E} \times \mathcal{E} = \sum_i \left(\mathbb{O}^+_{i,J=3,j=0} + \mathbb{O}^+_{i,J=3,j=2} + \mathbb{O}^+_{i,J=3,j=4} \right) + \sum_{n,i} \mathcal{D}_{2n} \mathbb{O}^+_{i,J=3+2n,j=4}$$

$T_{\mu\nu} \times T_{\rho\sigma}$

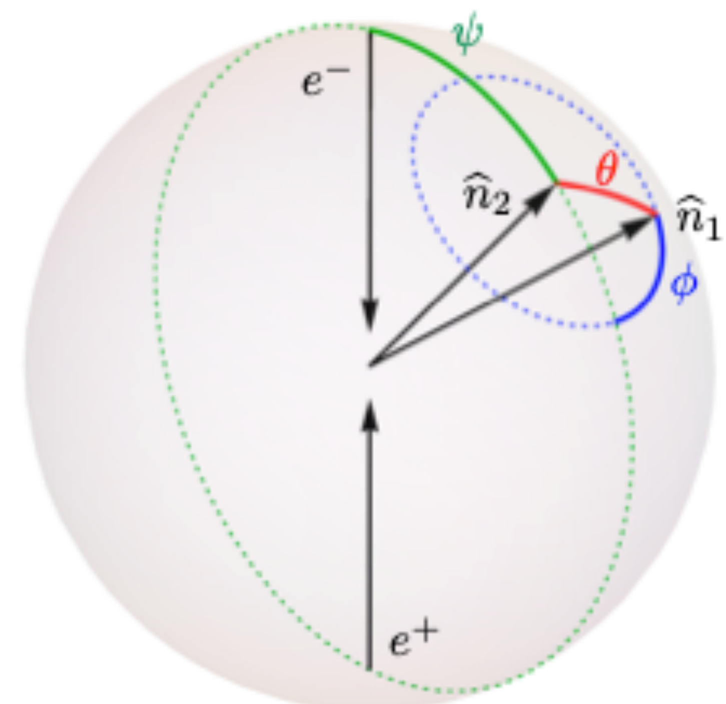


How to detect transverse spin in the experiment?

Oriented event shapes

$$e^+e^- \to \gamma^* \to \text{hadrons}$$

$$\epsilon^{*\nu}\langle J_\nu(p)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|J_\mu(p)\rangle\epsilon^\mu$$



1. Spinning targets

- Average over polarizations
- Do not average over the beam direction!

$$\theta^{\Delta_i-7}\left(\frac{1}{3}-\cos^2\phi\sin^2\psi\right)+\dots \qquad (j=2, l=2),$$

$$\theta^{\Delta_i-7}\left(\sin^2\psi-\frac{2}{3}\right)+\dots \qquad (j=0, l=2),$$

$$\theta^{\Delta_i-7}+\dots \qquad (j=0, l=0).$$

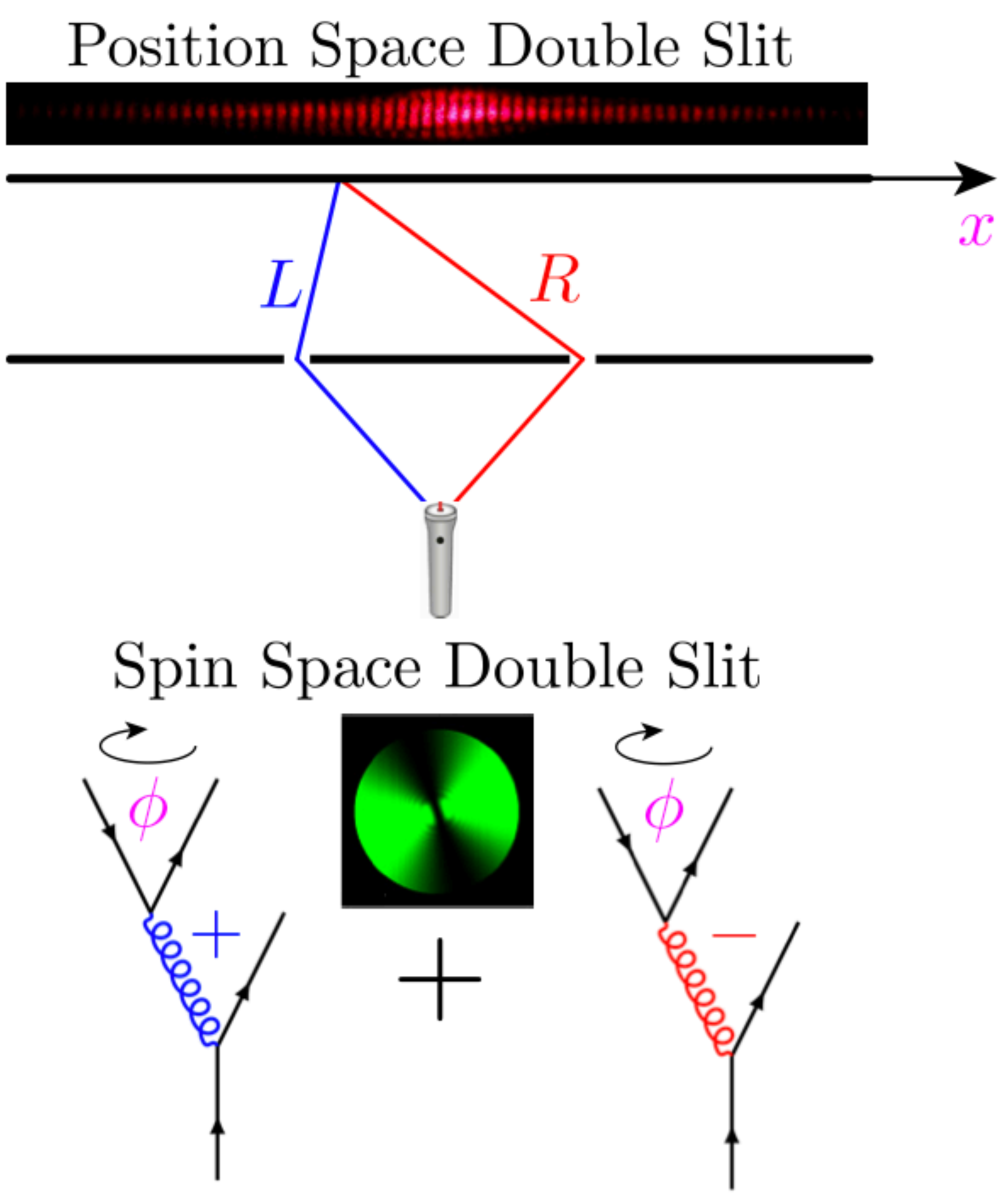
l = spin of the target density matrix

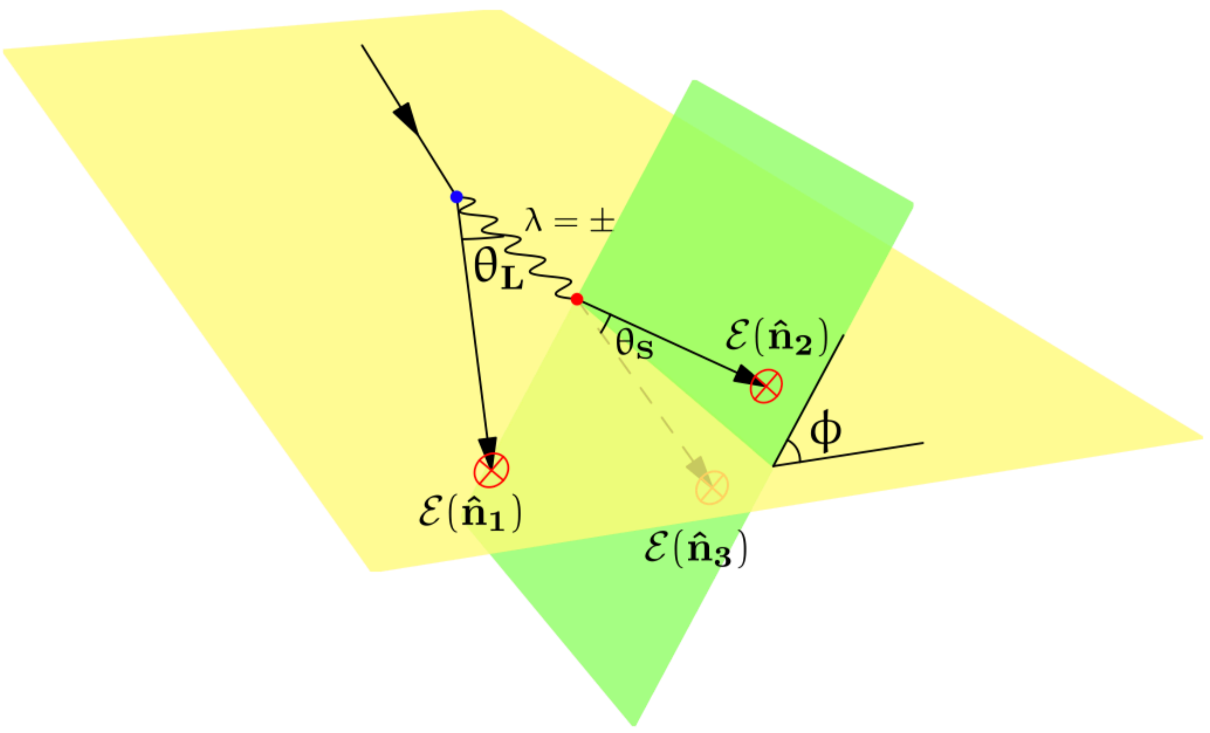
$$\boxed{j \le l}$$

2. Multi-point (three- or higher) energy correlators

Three-point event shapes

Double slit experiment in spin space and transverse spin.



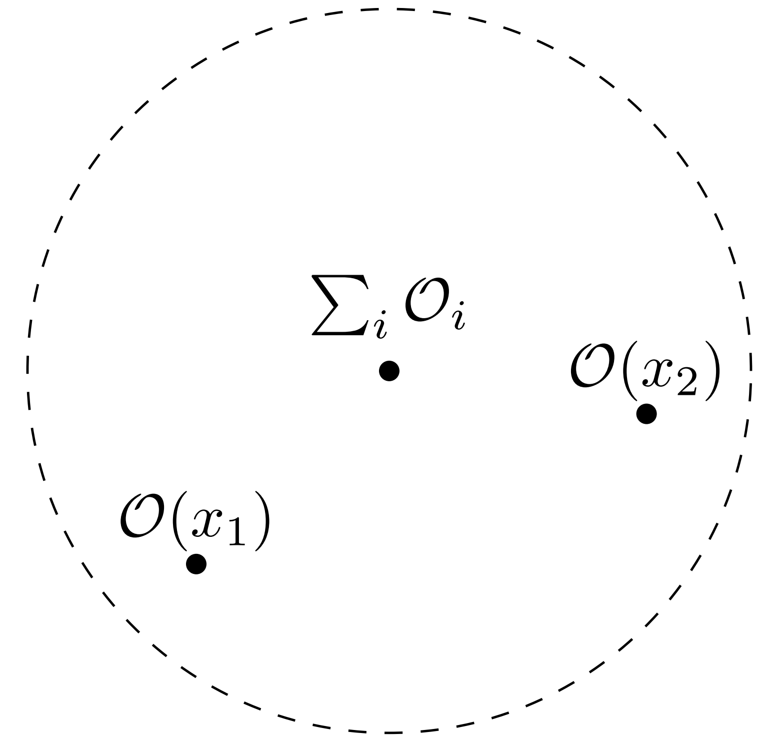


[Chen, Moult, Zhu '20]

Nature of the OPE

The usual argument

$$\mathcal{O}_1\mathcal{O}_2|\Omega\rangle = |\Psi\rangle = \sum_i |\mathcal{O}_i\rangle$$
$$|\mathcal{O}_i\rangle = \mathcal{O}_i|\Omega\rangle$$



Complete set of local operators.

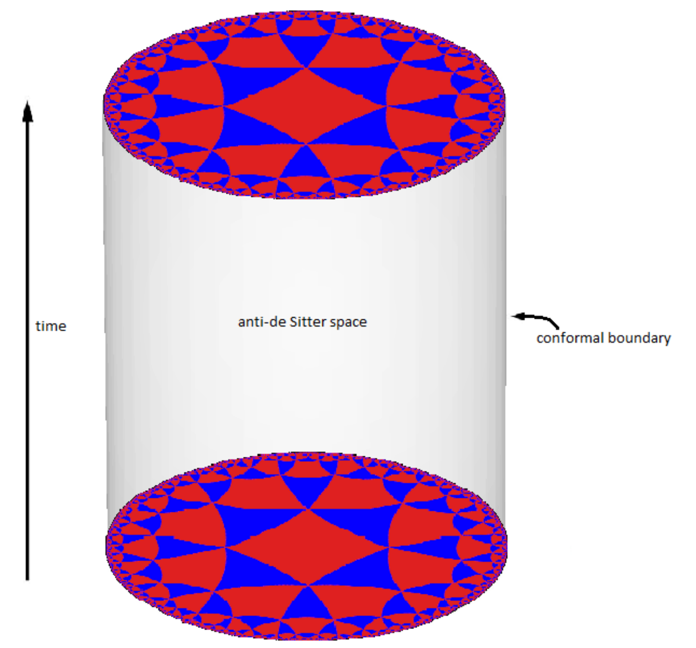
No such construction is available for light-ray operators.

The formula is derived by doing harmonic analysis on the celestial sphere, matching the kernel, and **closing the contour**.

$$\mathcal{D}_n \int' d^dx_1 d^dx_2 K^t_{\delta+1,J,j}(x_1,x_2;x,z)\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)$$
$$\underset{J\to J_1+J_2-1+n}{\longrightarrow} \int D^{d-2}z_1 D^{d-2}z_2 k_{\delta,j+n}(z_1,z_2;z)\mathbf{L}[\mathcal{O}_1](x,z_1)\mathbf{L}[\mathcal{O}_2](x,z_2).$$

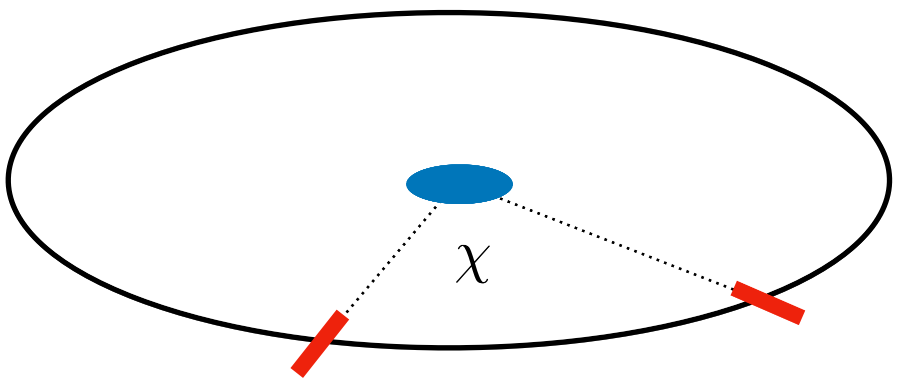
Superconvergence and gravitational EFTs

What about gravity dual?



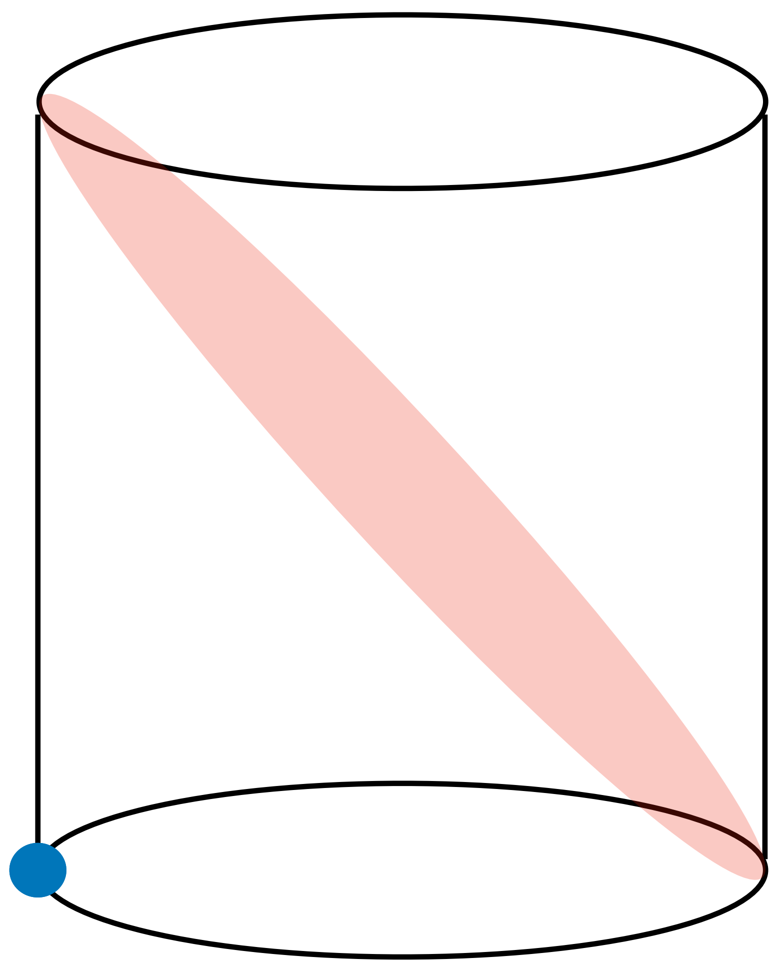
Event shape dual

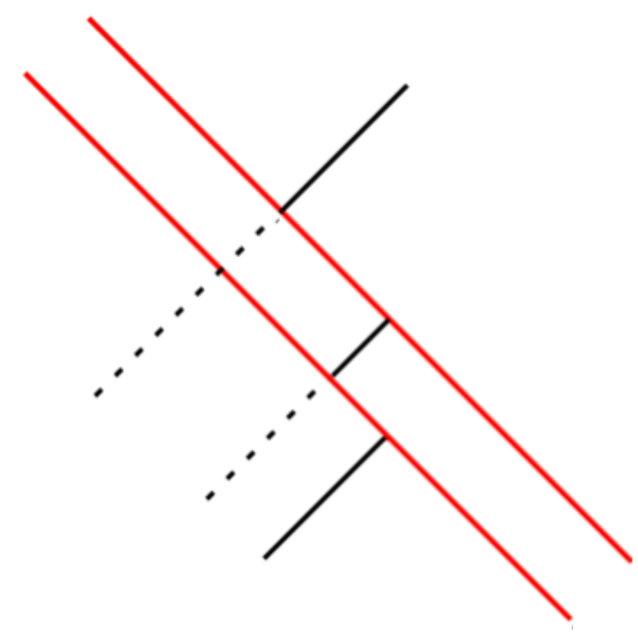
[Hofman, Maldacena 08’]



Quantum state at strong coupling.

Energy calorimeter	=	Gravitational shockwave
Measuring energy	=	Shapiro time delay





[Shapiro ’64]

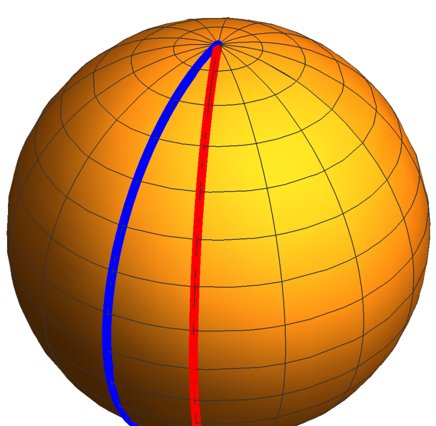
Conformal collider bounds

ANEC:

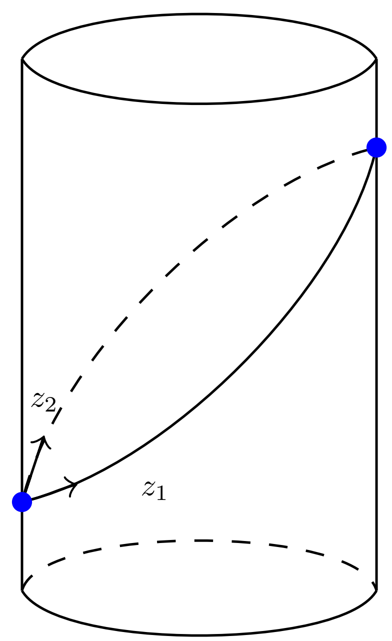
$$\mathcal{E}(\vec{n}) \geq 0$$

Superconvergence:

$$[\mathcal{E}(\vec{n}_1), \mathcal{E}(\vec{n}_2)] = 0$$



Bound on the Regge limit

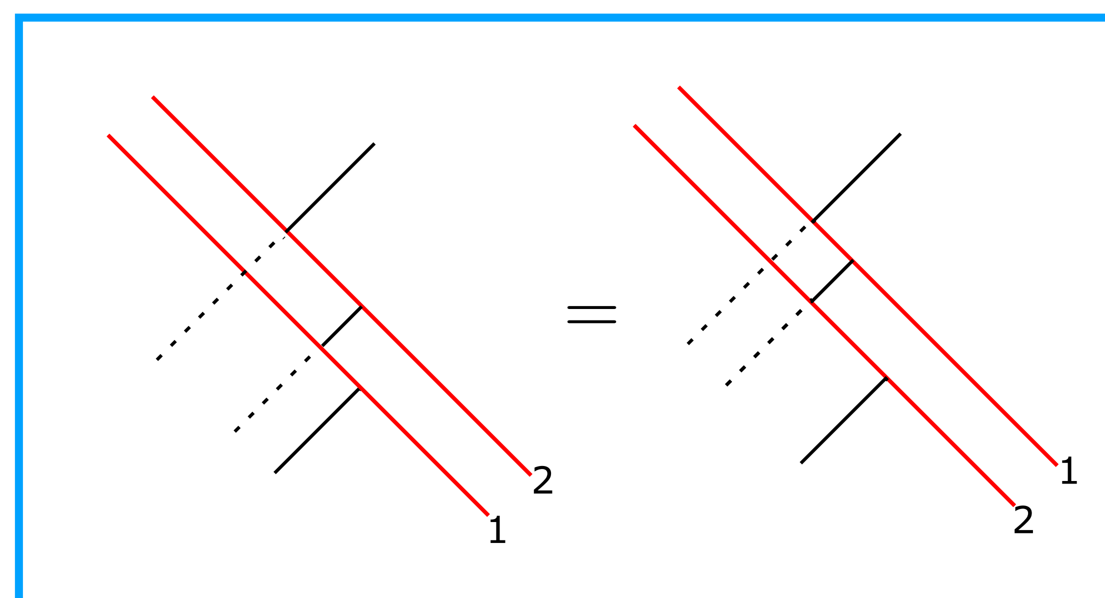


[Belin, Hofman, Mathys '19]
[Kologlu, Kravchuk, Simmons-Duffin, AZ '19]

Causality:

$$\Delta t_{\text{Shapiro}} \geq 0$$

Commutativity of shocks:
(stringy equivalence principle)



Bounds on gravitational EFTs

$$S = \frac{1}{l_{Pl}^{D-2}} \int d^5 x \sqrt{-g} \left(-2\Lambda + R + \alpha_{GB} \text{“}R^2\text{”} + \ldots\right)$$

Let us introduce a non-minimal coupling: $\alpha_{GB} \sim \frac{a-c}{c}$

ANEC:

$$\langle T \cdot \epsilon^*(p) | \mathcal{E}(\vec{n}) | T \cdot \epsilon(p) \rangle \geq 0 \quad \Rightarrow \quad -\frac{2}{3} \leq \frac{a-c}{c} \leq \frac{13}{18}$$

[Hofman, Maldacena '08]

Superconvergence:

$$\langle T \cdot \epsilon^*(p) | [\mathcal{E}(\vec{n}_1), \mathcal{E}(\vec{n}_2)] | T \cdot \epsilon(p) \rangle = 0 \quad \Rightarrow \quad \frac{a-c}{c} \sim \frac{1}{m_{\text{string}}^2}$$

$$\int dt \, \text{Disc}_t A(s,t) = 0$$

superconvergence

[Alfaro, Fubini, Rossetti, Furlan 66']

[Camanho, Edelstein, Maldacena, AZ '14]
[Kologlu, Kravchuk, Simmons-Duffin, AZ '19]

Conclusions

- New Lorentzian methods in CFTs
  - light-ray operators/event shapes
  - dispersion relations/sum rules

(conformal differential operators
conformal integral transforms, etc)

- More conceptually
  - what is the space of light-ray operators? (celestial bootstrap)
  - non-conformal theories (connection to QCD)
  - dispersive methods in quantum gravity (swampland vs landscape) flat space/dS?

- Experimentally
  - event shapes in the lab? (does the 3d ising model/QCP have jets?)
  - oriented and multi-point event shapes (higher transverse spin)